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Four-Dimensional $\mathcal{N}=1$ Supersymmetric Yang-Mills Theory on the Lattice without Fine-Tuning

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Abstract

We propose a method to formulate four-dimensional $\mathcal{N}=1$ super Yang-Mills theory on the lattice without fine-tuning. We first show that four-dimensional Weyl fermion in a real representation, which is equivalent to Majorana fermion, can be formulated using the domain wall approach with an addition of a Majorana mass term only for the unwanted mirror fermion. This formalism has manifest gauge invariance. Fermion number conservation is violated only by the additional Majorana mass term for the mirror fermion and the violation is propagated to the physical fermion sector through anomalous currents. Due to this feature, the formalism, when applied to the gluino in the present case, ensures the restoration of supersymmetry in the continuum limit without fine-tuning, unlike the proposal by Curci and Veneziano.

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Supersymmetry is one of the most exciting topics in field theory. It is important in two respects. In phenomenology, supersymmetry is motivated as a natural solution to the gauge hierarchy. From purely field theoretical points of view, supersymmetry enables analytic study of nonperturbative aspects of field theories. On the other hand, lattice formalism has been a powerful tool to extract nonperturbative dynamics of field theories. It would be nice to use this formalism to explore the nonperturbative dynamics of supersymmetric theories.

In spite of much effort in this direction, most of the attempts seem to have failed in practice. A practical proposal, however, has been given by Curci and Veneziano [1]. They propose to give up manifest supersymmetry on the lattice, and instead, to restore it in the continuum limit. It has been shown that this can indeed be done for four-dimensional $\mathcal{N} = 1$ super Yang-Mills theory using the Wilson-Majorana fermion for the gluino and fine-tuning the hopping parameter to the chiral limit.

Using this proposal, some numerical simulations have been started [2]. Although recent developments in treating dynamical fermions help quite a lot, it is needless to say that it would be better if we could do without fine-tuning. Instead of fine-tuning a bare parameter to the chiral limit, we could impose chiral symmetry on the lattice theory. This is not easy in standard lattice formulations of fermions, though [3]. In the section 9 of Ref. [4], it has been suggested that the overlap formalism can be used as such a formulation. The method we propose provides an alternative way, which is more suited for numerical study.

There is a formulation [5,6] that preserves chiral symmetry for Dirac fermion using the domain wall formalism [7]. We would like to propose a Majorana version of it. What we consider directly is Weyl fermion in a real representation, which is equivalent to Majorana fermion in four dimensions.

In Ref. [8], a proposal for formulating anomaly-free chiral gauge theories has been given. Their idea can be summarized as follows. Try to formulate it following Kaplan's proposal [7] for formulating chiral gauge theory on the lattice. We consider an extra dimension and consider five-dimensional Dirac fermion, whose mass has a domain-wall type dependence on the fifth coordinate. On the domain wall, we have a chiral zero mode, which could be used to construct chiral gauge theory. When the extent in the extra dimension is finite, however, we encounter an anti domain wall on which an unwanted mirror fermion with opposite chirality appears. Their idea to remedy this situation is to add a four-fermi gauge-invariant term motivated by Eichten-Preskill [9], only for the mirror fermion in order to give it a mass of the order of the cutoff, and thus to make it decouple in the continuum limit.

We apply this idea to the present case. Now since the Weyl fermion is in a real representation, we can introduce a gauge-invariant Majorana mass term only for the mirror fermion that serves for the same purpose. Here the dynamical effect of the additional term is quite clear, as compared with the four-fermi interaction in Ref. [8], and therefore it does not raise any further subtle problems. Fermion number violation in the physical fermion sector comes only from anomalous currents which pick up the violation due to the additional Majorana mass term for the mirror fermion. Restating this feature in terms of Majorana fermion, which is equivalent to Weyl fermion in a real representation, we have chiral symmetry up to the anomaly. Using this formalism for the gluino, we can obtain four-dimensional $\mathcal{N}=1$ super Yang-Mills theory in the continuum limit without fine-tuning.

Let us define the model we propose. We consider five-dimensional Dirac fermion on the lattice using the Wilson fermion formalism [10]. We take the coordinate in the fifth direction to be $-L_5 \le x_5 \le L_5$. The boundary condition is taken to be periodic in the fifth direction and either periodic or anti-periodic in the other four directions. The action is given by

$$S = \sum_x [\bar{\Psi}(x)\gamma_a D_a \Psi(x) + m(x_5)\bar{\Psi}(x)\Psi(x) + \frac{1}{2}\bar{\Psi}(x)\triangle\Psi(x)], \tag{1}$$

where $\Psi(x)$ is a four-component spinor, and γ_a $(a=1,\cdots,5)$ are the gamma matrices in five dimensions. The mass $m(x_5)$ has the domain-wall type dependence on x_5: $m(x_5)=m_0$ for $1 \le x_5 \le L_5$ and $m(x_5) = -m_0$ for $-L_5+1 \le x_5 \le 0$. The D_a and $\triangle$ are defined as follows.

$$D_a = \frac{1}{2}(\nabla_a^* + \nabla_a), \tag{2}$$

$$\triangle = \nabla_a^* \nabla_a, \tag{3}$$

where

$$\begin{aligned} \nabla_a \Psi(x) &= U_a(x)\Psi(x+\hat{a}) - \Psi(x), \\ \nabla_a^* \Psi(x) &= \Psi(x) - U_a^\dagger(x-\hat{a})\Psi(x-\hat{a}). \end{aligned} \tag{4}$$

The system is invariant under the gauge transformation.

$$\begin{aligned} \Psi(x) &\to g(x)\ \Psi(x), \\ \bar{\Psi}(x) &\to \bar{\Psi}(x)\ g^\dagger(x), \\ U_a(x) &\to g(x)\ U_a(x)\ g^\dagger(x+\hat{a}). \end{aligned} \tag{5}$$

The link variables $U_a(x)$ are taken such that $U_5(x) = I$ and $U_\mu(x)$ ($\mu = 1, \cdots, 4$) are independent of x_5.

In the following we take the following representation for the gamma matrices.

$$\gamma_\mu = \begin{pmatrix} 0 & \bar{\sigma}_\mu \\ \sigma_\mu & 0 \end{pmatrix} \qquad for\ \mu = 1, \cdots, 4; \qquad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \tag{6}$$

where $\sigma_\mu = (1, i\sigma_i)$, $\bar{\sigma}_\mu = (1, -i\sigma_i)$ and σ_i are the Pauli matrices. Now we decompose the four-component spinor $\Psi(x)$ as

$$\Psi = \begin{pmatrix} \psi \\ \chi \end{pmatrix}; \qquad \bar{\Psi} = (\, \bar{\psi} \quad \bar{\chi} \,)\, \gamma_0 = (\, \bar{\chi} \quad \bar{\psi} \,)\, . \tag{7}$$

We obtain

$$\begin{aligned} S = \sum_x [&\bar{\psi}(x)\sigma_\mu D_\mu \psi(x) + \bar{\chi}(x)\bar{\sigma}_\mu D_\mu \chi(x) \\ &+ \bar{\chi}(x) D_5 \psi(x) - \bar{\psi}(x) D_5 \chi(x) \\ &+ m(x_5)(\bar{\chi}(x)\psi(x) + \bar{\psi}(x)\chi(x)) + \frac{1}{2}(\bar{\chi}(x)\triangle\psi(x) + \bar{\psi}(x)\triangle\chi(x))]. \end{aligned} \tag{8}$$

Due to the argument in Ref. [7], we have a chiral zero mode in $\psi(x)$ localized around the first wall $x_5 = 0$, while we have another chiral zero mode in $\chi(x)$ localized around the second wall $x_5 = L_5$. There are many other massive modes, whose contribution has to be appropriately canceled by additional five-dimensional massive scalar fields as in [5], which we omit in this paper. We can see that the theory has exact chiral symmetry in the $L_5 \to \infty$ limit by rewriting it in terms of the overlap formula using the transfer matrix formalism as has been done in Ref. [11]. $|0\pm\rangle$ are defined as the ground states of the transfer matrices $\hat{T}_\pm$ in the fifth direction, whose explicit forms are given by eq.(2.12) of Ref. [11]. $\pm$ correspond to the positive m region and the negative m region, respectively. The fermion determinant can be expressed as $\langle 0-|0+\rangle\langle 0+|0-\rangle$. The two overlaps correspond to the chiral fermions on the two walls. When the gauge configuration is topologically trivial, $|0\pm\rangle$ have the same fermion number and the fermion number is conserved separately on the two walls. On the other hand, when the gauge configuration is topologically non-trivial, $|0\pm\rangle$ have different fermion numbers and the fermion number is violated on each wall, though conserved as a whole. Thus we have exact chiral symmetry up to the anomaly. Note that the symmetry cannot be seen in the action (8). This is because the $L_5 \to \infty$ limit is essential in obtaining the exact symmetry. When L_5 is finite, the chiral symmetry is broken due to the non-vanishing overlap of the wave-functions of the chiral zero modes on the two walls. However, this is expected to be exponentially small for sufficiently large L_5 and thus we have almost exact chiral symmetry.

Now let us consider the chiral zero mode on the first wall $x_5 = 0$ as the physical Weyl fermion and the one on the second wall $x_5 = L_5$ as the unwanted mirror fermion. In order to give the mirror fermion a mass, we add a Majorana mass term to the action localized on the second wall.

$$S_{add} = M \sum_x [\chi^T(x)\sigma_2\chi(x) + \bar{\chi}(x)\sigma_2\bar{\chi}^T(x)]\Big|_{x_5=L_5} , \tag{9}$$

where M is kept fixed when one takes the continuum limit. Note that this term is gauge invariant when the fermion is in a real representation of the gauge group, since we have $g^T(x)\ g(x) = 1$. Note also that it is invariant under four-dimensional rotation and translation. Let us see what we obtain in the $L_5 \to \infty$ limit. Now the fermion determinant can be written as $\langle 0-|0+\rangle\langle 0+|\mathcal{O}|0-\rangle$, where $\mathcal{O}$ is an operator which violates fermion number. Hence the chiral fermion on the second wall acquires Majorana mass of the order of the cutoff, while the chiral fermion on the first wall remains massless. The fermion number of the chiral fermion on the first wall is conserved up to the anomaly. The violation of the symmetry when the L_5 is finite is expected to be exponentially small as before.

Thus we can formulate four-dimensional chiral gauge theory with Weyl fermion in a real representation using the domain wall approach with an addition of the Majorana mass term for the unwanted mirror fermion. This is not so surprising, because Weyl fermion in a real representation in four dimensions is equivalent to Majorana fermion and thus the theory is essentially vector-like. However, the important feature of this formalism, as compared with the Wilson fermion formalism for the Majorana fermion, is that the theory has the chiral symmetry up to the anomalous currents propagating from the mirror fermion sector. Due to this feature, we can use the formalism for a gauge theory with massless Majorana fermion in the adjoint representation, which is expected to be supersymmetric in the continuum limit without fine-tuning.

In the large N limit of the gauge group, there is yet another possibility to deal with supersymmetry as was suggested in Ref. [12]. We might be able to use the continuum version of the Eguchi-Kawai model. The advantage of this approach is that it has manifest supersymmetry. Indeed there is a revived interest in this model in the context of superstring theory [13]. In contrast to the lattice version of the Eguchi-Kawai model [14], which is known to be equivalent to the infinite volume lattice theory, in the continuum version, the connection to the infinite volume continuum theory is not so obvious beyond perturbation theory. It is therefore worth while to study the large N limit of super Yang-Mills theory using our method. For this purpose, the reduction of dynamical degrees of freedom in the large N limit [14] is essential. Since our formalism is an ordinary Lagrangian field theory

with all the fields in the adjoint representation, we can perform the twisted reduction procedure [15] for the four-dimensional space-time directions. Thus, our model in the large N limit is equivalent to the reduced model in which the space-time direction is reduced, while the extra direction is left unreduced. When one uses the overlap formalism [4] to deal with super Yang-Mills theories without fine-tuning, the large N reduction is not obvious. This is another advantage of the present approach.

To summarize, we have proposed a method to deal with four-dimensional $\mathcal{N} = 1$ super Yang-Mills theory on the lattice without fine-tuning. A formalism for dealing with Majorana fermion with chiral symmetry up to the anomaly has been given. Using this formalism for the gluino, the supersymmetry is expected to be restored in the continuum limit without fine-tuning, which makes studies in this direction much more efficient. One of the most interesting physics accessible with the present method is the gluino condensation.

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